RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2016-19]
B.A./B.Sc. SECOND SEMESTER (January – June) 2017
Mid-Semester Examination, March 2017

Date : 15/03/2017
PHYSICS (Honours)
Time : 11 am– 1 pm
Paper : II
Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

(Answer any three questions taking at least one from each unit) **[3×10]**

Unit - I

1. a) Find the Fourier series of [3]

$$f(x)=\begin{cases}0 & -2\le x\le 0\\ x & 0\le x\le 2\end{cases}$$

b) Find the Fourier transform of e^{-ax^2} where a > 0 [3]

c) What is Dirac Delta function. Show that $\lim\limits_{\alpha\to\infty}\frac{\alpha}{\sqrt{\pi}}e^{-\alpha^2x^2}$ is a Dirac Delta function. [1+3]

2. a) Find the general solution of Laplace's equation $\nabla^2\phi=0$ in 3-dimensional spherical polar coordinates. [8]

b) Assume there is an axial symmetry along z-axis, how the general solution will reduce (i) inside and (ii) outside a solid sphere of radius r. [2]

Unit - II

3. a) Find the gravitational potential and gravitational field in different regions of a thick spherical shell of inner radius a and outer radius b. [6]

b) Define axial modulus. Show that axial modulus χ holds the relation $\chi=K+\frac{4}{3}n$ where K is the bulk modulus and n is the modulus of rigidity. [4]

4. a) A particle of mass m moves under a central force. Show that,

i) total mechanical energy is conserved.

ii) the angular momentum I about the centre of force is a constant of motion. What does it imply for the nature of the orbit? [4]

b) If the force is of the form, $\vec{F}=-\hat{r}\frac{k}{r^2},(k>0)$, Show that the vector $\vec{A}=\vec{p}\times\vec{L}-mk\hat{r}$ is also a constant of motion. Calculate the magnitude of $\vec{A}$. [3]

c) The equation of a central-force orbit is given by $r=Ae^{k\theta}$, where A, k are constants. Find (i) the force and (ii) the time-dependence of θ. [3]

5. a) A particle of mass m and total energy E is moving in the force field $f(r)=-\frac{k}{r^2}(h>0)$ in an elliptic orbit, $\frac{\ell}{r}=1+e\cos\theta$ (symbols are standard)

Show that its total energy is given by $E=-\frac{k}{2a}$, where a is the semi-major axis of the ellipse. [5]

b) Find the maximum and minimum velocities, V_{max} and V_{min}, respectively of the particle in the above orbit. Show that $\frac{V_{max}}{V_{min}} = \frac{1+e}{1-e}$. [5]

Group – B

(Answer <u>any two</u> questions from <u>question nos. 6 to 8</u>) **[2×10]**

6. a) Determine the speed and the direction of propagation of the travelling wave :
$\psi(x,t) = 5\cdot 0\exp(-ax^2 - bt^2 - 2\sqrt{ab}xt)$ here a = 25 m^{-2} and b = 9s^{-2}. [3]

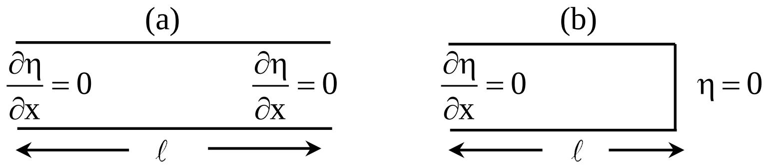
b) Discuss the distinction between phase and group velocities. Derive an expression for the relation between them. [2+2]

c) The refractive index of a certain glass is given by the formula : $n = A + B/\lambda_0^2$ where λ_0 is the wavelength in vacuum. Show that the ratio of the group velocity to the phase velocity is $(A\lambda_0^2 + B)/(A\lambda_0^2 + 3B)$. [3]

7. a) Prove that the expression for the speed of a longitudinal wave in an ideal gas may be written $v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_a}$. where a denotes reversible adiabatic process.
[**Hint :** You may directly start from a known expression for v] [2]

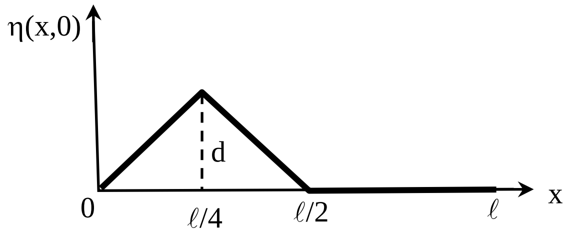
b) Hi-fi music system is played very loudly at an intensity of $100I_0(I_0 = 10^{-2}W/m^2$ being the commonly used standard of sound intensity) in a small room of cross-section 3m × 3m. Show that the audio power output is about 10W. [2]

c) Standing acoustic waves are formed in a tube of length l with (a) both ends open and (b) one end open and the other closed. If the particle displacement : $\eta = (A\cos kx + B\sin kx)\sin\omega t$



and the boundary conditions are as shown in the diagrams, show that for
(a) (both ends open) : $\eta = A\cos kx\sin\omega t$ with $\lambda = 2\ell/n$. and for (b) (one end open, the other closed) : $\eta = A\cos kx\sin\omega t$ with $\lambda = 4\ell/(2n+1)$. Sketch the first three harmonics (only the x-dependent term) for each case. [2+2+2]

8. a) A string, clamped at x = 0 and at x = ℓ, is plucked at $x = \ell/4$ but stopped from the center to the end, so that the initial displacement is given by



$$\eta(x,0) = \frac{4d}{\ell}x, \text{for } 0 < x < \frac{\ell}{4}$$
$$= \frac{4d}{\ell}\left(\frac{\ell}{2} - x\right), \text{for } \frac{1}{4} < x < \frac{\ell}{2}$$
$$= 0, \text{ for } \frac{1}{2} < x < 1$$

The initial velocity, $\left(\frac{\partial \eta}{\partial t}\right)_{t=0} = 0$ obviously.

Assuming a solution to the wave equation for the transverse vibrations of this plucked string of the form: $\eta(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{\ell}(C_n \cos\omega_n t + D_n \sin\omega_n t)$, find the coefficients (C_n and / or D_n) satisfying the given initial conditions. Therefore, write the (Fourier) series representing $\eta(x,t)$. [If you are unable to write the series in a general summation form then try to write it explicitly, term by term (at least first 6/7 terms).] [6]

b) Show that, in the Doppler effect, the change of frequency noted by a stationary observer O as a moving source S' passes him (assume the angle between them to be zero) is given by

$$\Delta v = \frac{2vcu}{c^2 - u^2}$$

where c is the wave velocity and u is the velocity of S'. [2]

c) Light from a star of wavelength 6×10^{-7}m is found to be shifted 10^{-11}m towards the red when compared with the same wavelength from a laboratory source. If the velocity of light is $3\times10^{8}\,m/s$ show that the earth and the star are separating at a velocity of 5 Km/s. [2]

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